The Principles of School Mathematics

- Highlight the basic characteristics of a high-quality mathematics instructional program and provide guidance for making educational decisions

The Equity Principle- Excellence in mathematics education requires equity—high expectations and strong support for all students.

The Teaching Principle- Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.

The Learning Principle- Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.

The Assessment Principle- Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.

The Curriculum Principle- A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.

The Technology Principle- Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.

What are the Standards for School Mathematics?

- The ten standards in ***Principles and Standards for School Mathematics*** describe the mathematical knowledge, understanding, and skills that students should acquire from prekindergarten through grade 12.

The Content Standards

- Number and Operations
- Algebra
- Geometry
- Measurement
- Data Analysis and Probability

The Process Standards

- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representations

Problem Solving

, Instructional programs from prekindergarten through grade 12 should enable all students to-

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.

Problem solving means engaging in a task for which the solution method is not known in advance. In order to find a solution, students draw on their knowledge, and through this process, they will often develop new mathematical understandings. Solving problems is not only a goal of learning mathematics but also a major means of doing so. Students should have frequent opportunities to formulate, grapple with, and solve complex problems that require a significant amount of effort and should then be encouraged to reflect on their thinking.

By learning problem solving in mathematics, students should acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the mathematics classroom. In everyday life and in the workplace, being a good problem solver can lead to great advantages!

Reasoning and Proof

, Instructional programs from prekindergarten through grade 12 should enable all students to-

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs;
- select and use various types of reasoning and methods of proof.

Mathematical reasoning and proof offer powerful ways of developing and expressing insights about a wide range of phenomena. People who reason and think analytically tend to note patterns, structure, or regularities in both real-world situations and symbolic objects; they ask if those patterns are accidental or if they occur for a reason; and they conjecture and prove. Ultimately, a mathematical proof is a formal way of expressing particular kinds of reasoning and justification.

Being able to reason is essential to understanding mathematics. By developing ideas, exploring phenomena, justifying results, and using mathematical conjectures in all areas and–with different expectations of sophistication–at all grade levels, students should see and expect that mathematics makes sense.

Communication

, Instructional programs from prekindergarten through grade 12 should enable all students to-

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.

Communication is an essential part of mathematics and mathematics education. It is a way of sharing ideas and clarifying understanding. Through communication, ideas become objects of reflection, refinement, discussion, and amendment. The communication process also helps build meaning and permanence for ideas and makes them public. When students are challenged to think and reason about mathematics and to communicate the results of their thinking to others orally or in writing, they learn to be clear and convincing. Listening to others' explanations gives students opportunities to develop their own understandings. Conversations in which mathematical ideas are explored from multiple perspectives help the participants sharpen their thinking and make connections. Students who are involved in discussions in which they justify solutions-especially in the face of disagreement-will gain better mathematical understanding as they work to convince their peers about differing points of view (Hatano and Inagaki 1991). Such activities also help students develop a language for expressing mathematical ideas and an appreciation of the need for precision in that language.

Connections

, Instructional programs from prekindergarten through grade 12 should enable all students to-

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
- recognize and apply mathematics in contexts outside of mathematics.

When students can connect mathematical ideas, their understanding is deeper and more lasting. They can see mathematical connections in the rich interplay among mathematical topics, in context that relate mathematics to other subjects, and in their own interests and experience. Through instruction that emphasizes the interrelatedness of mathematical ideas, students not only learn mathematics, they also learn about the utility of mathematics.

Mathematics is not a collection of separate strands or standards, even though it is often partitioned and presented in this manner. Rather, mathematics is an integrated field of study. Viewing mathematics as a whole highlights the need for studying and thinking about the connections withing the discipline, as reflected both within the curriculum of a particular grade and between grade levels. To emphasize the connections, teachers must know the needs of their students as well as the mathematics that the students studied in the preceding grades and what they will study in the following grades. As the Learning Principle emphasizes, understanding involves making connections.

Representation

, Instructional programs from prekindergarten through grade 12 should enable all students to-

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.

The ways in which mathematical ideas are represented is fundamental to how people can understand and use those ideas. Consider how much more difficult multiplication is using Roman numerals (for those who have not worked extensively with them) than using Arabic base-ten notation. Many of the representations we now take for granted- such as numbers expressed is base-ten or binary form, fractions, algebraic expressions and equations, graphs, and spreadsheet displays- are the result of a process of cultural refinement that took place over many years. When students gain access to mathematical representations and the ideas they represent, they have a set of tools that significantly expand their capacity to think mathematically.

The term *representation* refers both to process and to product-in other words, to the act of capturing a mathematical concept or relationship in some form and to the form itself. Students represent ideas when they create a table of data about weather patterns, when they describe in words or with a picture the important features of an object such as a cylinder, or when they translate aspects of a problem into an equation. Good representations fulfill a dual role: they are tools for thinking and instruments for communicating.